MATHEMATICAL MODELS, TABLES AND NOMOGRAMS TO SETTLE THE TECHNICALLY OPTIMAL RATES (TOR) OF N, P\textsubscript{2}O\textsubscript{5} AND K\textsubscript{2}O IN WINTER WHEAT. COMPARISONS WITH SOME WRONG MODELS PUBLISHED BY D. AND V. DAVIDESCU, R. MADJAR, G. NEAȚĂ

GH. C. BUDOI

University of Agronomic Sciences and Veterinary Medicine of Bucharest

**Keywords:** nutrient rates, maximum yield, sustainable agriculture

**Abstract**

This is the first paper from the series concerning the field crops. The mathematical models and agrochemical tables and nomograms in winter wheat are presented and these serve to settle the Technically Optimal Rates (TOR) of N, P\textsubscript{2}O\textsubscript{5} and K\textsubscript{2}O as function of maximum expected yield, \(Y_m\), and the specific soil agrochemical indexes, AI (nitrogen index: IN; mobile phosphorous: \(P_{Alc}\); mobile potassium: \(K_{Al}\)). The TOR of a nutrient, N for example, is that rate which allows to obtain the maximum yield in given conditions concerning the soil supply with the regarded nutrient, and allow a sustainable agriculture. The TOR system uses the same equations for nutrients action coefficients and soil nutrient supply as the Economically Optimal Rate (EOR) system. Fertexpert software version 3 has been used for TOR calculations. When the exact values of TOR are desired, the mathematical model have to be used; when operative settle of TOR are desired, the practical agrochemical tables or nomograms can be used by the farmers. The paper presents comparison between the nutrient rates calculated with TOR model and the absurd rates calculated with some published erroneous models.

**INTRODUCTION**

The TOR of N, P\textsubscript{2}O\textsubscript{5} or K\textsubscript{2}O are those rates that allow to obtain the maximum yield in given conditions concerning the soil supply with the considered nutrient, while the Economically Optimal Rates, EOR, allow to obtain the maximum net revenue/ha. The advantages and disadvantages of TOR system [3] versus EOR system [1] have been recently presented in another paper [4]. To can establish TOR is of crucial importance for the crop technologies. Unfortunately, many published mathematical models are wrong and give absurd nutrient rates [6, 7, 8, 9, 10]. This paper brings, for the first time, reasonable solutions in settling TOR in winter wheat depending on the yield level and relevant soil agrochemical indexes.

**MATERIAL AND METHODS**

A new version of FERTEXPERT software [2], the 3-rd, have been developed by author in order to incorporate the mathematical TOR model [3] and to use the
specific parameters for winter wheat. The mathematical models used by software for TOR, nutrient action coefficients, $c_a$, and soil nutrient supply, $S_n$, are presented below; the equations for $c_a$ and $S_n$ have the same coefficients as those used for the EOR of N, P$_2$O$_5$ and K$_2$O [1]. FERTEXPERT has then been used for TOR calculations. The specific practical agrochemical tables and nomograms (graphs) have been built based on the calculated TOR.

RESULTS AND DISCUSSION

1. Specific results and discussion concerning the TOR in winter wheat. The mathematical model, logically derived from the modified Mitscherlich response function [3], which can be used in order to calculate the TOR, is:

$$
TOR, \text{kg N, P}_2\text{O}_5, \text{K}_2\text{O/ha} = \frac{\log(2.3\cdot c_a \cdot Y_m)}{c_a - S_n}$$

(condition: if $\frac{\log(2.3 \cdot c_a \cdot Y_m)}{c_a - S_n} < 0$, then $TOR = 0$)

where: $Y_m =$ maximum expected yield, kg/ha, established on the basis of the site evaluation studies; the level of $Y_m$ depend on the variety and on the levels of all vegetation factors (temperature, water etc.); $c_a =$ action coefficient of N, P, K, unique for the nutrient from fertilizer and for the nutrient from soil (potentially available form), calculated with specific models; $S_n =$ soil nutrient supply, potentially available form, kg N, P$_2$O$_5$ or K$_2$O/ha, calculated based on the specific agrochemical analyses (IN, P$_{ALc}$, K$_{AL}$) and $Y_m$. The logical deduction of the TOR mathematical model has been demonstrated by author in another paper [3].

The same equations and coefficients as those of EOR system [1] are used in order to compute $c_a$ and $S_n$ in TOR system; these equations are [1]:

For nitrogen: $c_a = 0.006 + 12/Y_m; \quad S_n = 24.5IN - 2.41(IN)^2 + 0.0015Y_m$

For phosphorus: $c_a = 0.004 + 14/Y_m; \quad S_n = 137(1 - 10^{-0.018P_{ALc}}) + 0.0045Y_m$

For potassium: $c_a = 0.0047 + 17.5/Y_m; \quad S_n = 180(1 - 10^{-0.00362K_{AL}}) + 0.003Y_m$

In these models: IN = soil nitrogen index; P$_{ALc}$ = soil mobile P content, ppm P, corrected with a reaction factor, FR [1]; K$_{AL}$ = soil mobile K content, ppm K.

The calculated results concerning the TOR values depending on the relevant soil agrochemical indexes, AI (IN, P$_{ALc}$, K$_{AL}$) and on the maximum expected yield, $Y_m$, are presented in two-sided tables (tables 1, 2, 3) and in 3 nomograms (figure 1). These agrochemical tables and nomograms show that the TOR increase with the decrease of AI and with the increase of $Y_m$. When the exact values of TOR are desired, the TOR mathematical model has to be used. When operative settle of TOR are desired, the practical agrochemical tables or nomograms can be used by the farmers; in such cases, visual interpolations have to be done, if necessary, in the TOR estimations; when interpolations have to be done, the results are approximates. The interpolations using a nomogram are more precise than those using a table. The agrochemical tables and nomograms in winter wheat, published
for the first time in this paper, will be accessible on a specific website through an online Decision Support System (DSS).

**Table 1**

<table>
<thead>
<tr>
<th>(Y_m) (kg/ha)</th>
<th>IN</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>130</td>
<td>120</td>
<td>110</td>
<td>102</td>
<td>96</td>
<td>90</td>
<td>85</td>
<td>82</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>167</td>
<td>157</td>
<td>148</td>
<td>140</td>
<td>133</td>
<td>127</td>
<td>123</td>
<td>120</td>
<td>117</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>195</td>
<td>185</td>
<td>175</td>
<td>167</td>
<td>160</td>
<td>155</td>
<td>150</td>
<td>147</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>217</td>
<td>206</td>
<td>197</td>
<td>189</td>
<td>182</td>
<td>177</td>
<td>172</td>
<td>169</td>
<td>167</td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>234</td>
<td>224</td>
<td>215</td>
<td>207</td>
<td>200</td>
<td>194</td>
<td>190</td>
<td>187</td>
<td>184</td>
<td></td>
</tr>
<tr>
<td>7000</td>
<td>249</td>
<td>238</td>
<td>229</td>
<td>221</td>
<td>214</td>
<td>209</td>
<td>204</td>
<td>201</td>
<td>199</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>(Y_m) (kg/ha)</th>
<th>ppm P</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>120</td>
<td>99</td>
<td>82</td>
<td>68</td>
<td>57</td>
<td>48</td>
<td>41</td>
<td>35</td>
<td>30</td>
<td>26</td>
<td>20</td>
<td>16</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>3000</td>
<td>165</td>
<td>145</td>
<td>128</td>
<td>114</td>
<td>103</td>
<td>94</td>
<td>86</td>
<td>80</td>
<td>75</td>
<td>71</td>
<td>65</td>
<td>62</td>
<td>59</td>
<td></td>
</tr>
<tr>
<td>4000</td>
<td>201</td>
<td>180</td>
<td>163</td>
<td>150</td>
<td>138</td>
<td>129</td>
<td>122</td>
<td>116</td>
<td>111</td>
<td>107</td>
<td>101</td>
<td>97</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>230</td>
<td>209</td>
<td>192</td>
<td>178</td>
<td>167</td>
<td>158</td>
<td>151</td>
<td>145</td>
<td>140</td>
<td>136</td>
<td>130</td>
<td>126</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>6000</td>
<td>254</td>
<td>233</td>
<td>216</td>
<td>202</td>
<td>191</td>
<td>182</td>
<td>174</td>
<td>168</td>
<td>163</td>
<td>159</td>
<td>154</td>
<td>150</td>
<td>147</td>
<td></td>
</tr>
<tr>
<td>7000</td>
<td>273</td>
<td>252</td>
<td>235</td>
<td>222</td>
<td>211</td>
<td>201</td>
<td>194</td>
<td>188</td>
<td>183</td>
<td>179</td>
<td>173</td>
<td>169</td>
<td>167</td>
<td></td>
</tr>
</tbody>
</table>

**Table 3**

<table>
<thead>
<tr>
<th>(Y_m) (kg/ha)</th>
<th>ppm K</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
<th>180</th>
<th>200</th>
<th>220</th>
<th>240</th>
<th>260</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000</td>
<td>76</td>
<td>56</td>
<td>39</td>
<td>25</td>
<td>13</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3000</td>
<td>116</td>
<td>96</td>
<td>80</td>
<td>65</td>
<td>53</td>
<td>43</td>
<td>35</td>
<td>27</td>
<td>21</td>
<td>16</td>
<td>12</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>4000</td>
<td>148</td>
<td>128</td>
<td>112</td>
<td>98</td>
<td>86</td>
<td>75</td>
<td>67</td>
<td>59</td>
<td>53</td>
<td>48</td>
<td>44</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>5000</td>
<td>175</td>
<td>155</td>
<td>138</td>
<td>124</td>
<td>112</td>
<td>101</td>
<td>93</td>
<td>86</td>
<td>79</td>
<td>74</td>
<td>70</td>
<td>66</td>
<td>66</td>
</tr>
<tr>
<td>6000</td>
<td>196</td>
<td>176</td>
<td>159</td>
<td>145</td>
<td>133</td>
<td>123</td>
<td>114</td>
<td>107</td>
<td>101</td>
<td>96</td>
<td>91</td>
<td>88</td>
<td>88</td>
</tr>
<tr>
<td>7000</td>
<td>214</td>
<td>194</td>
<td>178</td>
<td>163</td>
<td>151</td>
<td>141</td>
<td>133</td>
<td>125</td>
<td>119</td>
<td>114</td>
<td>110</td>
<td>106</td>
<td>106</td>
</tr>
</tbody>
</table>

2. Comparison between TOR and EOR and mistakes in the use of EOR model, tables and nomograms, avoided by TOR system. These aspects are presented in the complementary paper published by author in these Proceedings [5].

532
3. Comparison between the macronutrient rates in winter wheat calculated with the above TOR mathematical and those calculated with the erroneous models elaborated and published by some authors [6], [7], published by other authors [8], [9], [10] with the same or new errors. In literature, the mathematical models wear in texts the names of the authors which elaborated and published them. This was not allowed me by the Scientific Committee of this Proceedings.

3.1. Reasonable nutrient rates given by the above TOR mathematical model.
For example, on a soil with 35 ppm P, we have to apply 41 kg P$_2$O$_5$/ha for a 2000 kg/ha yield level and 122 kg P$_2$O$_5$/ha for a 4000 kg/ha yield level (table 2).

3.2. Absurd, huge nutrient rates given by the erroneous model

"D, kg/ha = (C$_o$ - R$_t$) · G$_{sa}$ · (1/C$_u$) · k"

elaborated and published by some authors [6], [7], published also by others [9], [10], where: D = the rate to be applied (nitrogen, phosphorous, potassium), kg/ha, to reach the optimum content in soil; C$_o$ = the optimum soil content which has to be attained in soil (nitrogen, phosphorous or potassium), ppm N, P, K (it is logic that C$_o$ assure maximum yield/ha); R$_t$ = total soil reserve of nitrogen, phosphorous or potassium (potentially available forms), in ppm; G$_{sa}$ = weight of soil arable layer, t/ha; C$_u$ = utilization coefficient of fertilizer, % (12-40 %); k = correction coefficient depending on soil organic matter content for N and K rates and on soil pH, texture and degree of gleization.
Some authors [9] published the above model even in a more absurd form, with Rt in t/ha, which means that we have to subtract t/ha from ppm.

Example of calculation: for Co = 80 ppm P; Rt = 35 ppm P; Gsa = 3000 t/ha, Cu = 12 %, k = 1, kg P₂O₅ = kg P·2.29, the rate is

\[ D = (80 - 35) \times 3000 \times \frac{1}{12} = 11250 \text{ kg P/ha} = 25762 \text{ kg P}_2\text{O}_5/\text{ha} \text{ (absurd)} = 143125 \text{ kg simple superphosphate/ha} \text{ (with 18 \% P}_2\text{O}_5) = 29 \text{ trucks} \text{ of 5 t/ha} \text{ (absurd)}. \]

3.3. Huge, absurd, nonsense negative rates of P₂O₅ given by the wrong model

"D_p (kg/ha) = \left( P_{ex} - P_t \times 100 \right) / C_u \"",
elaborated and published by some authors [6], published also by other authors [8], and published by another author [10] as

"D_p (kg/ha) = \left( P_{ex} - P_t \right) \times 100 / C_u \"",
where P_{ex} = phosphorous extracted with the yield (Y), in kg/ha: \( P_{ex} = Y \times C_{sp} \); \( C_{sp} \) = specific phosphorous consumption per tone of main yield: 15-18 kg P₂O₅/t [6]; \( C_u \) = utilization coefficient of fertilizer, %; \( P_t \) = total soil reserve of potentially available phosphorous, kg/ha, which, when we do not apply manure, is calculated with the equation \( P_t = G_{sa} \times P \times k_p \times C_{Ag} \times C_{pH} \times C_{fr} / 1000000 \); \( G_{sa} \) = weight of soil arable layer, kg/ha; \( P \) = mobile soil content, ppm P; \( k_p \) = coefficient of P assimilation from fertilizer (that means \( C_u \)); \( C_{Ag} \times C_{pH} \) = coefficient of assimilation depending on soil pH (in fact this is a correction factor of mobil soil P content with a reaction factor, \( C_{fr} [1] \), specific only for the AL method (extraction with ammonium acetate lactate), so the authors falsely consider this factor coefficient of P assimilation); \( C_{Ag} \) = coefficient of assimilation (0.6-1) depending on soil gleization degree.

Among other errors, the P_t equations do not contain the multiplication coefficient 2.29 in order to transform P in P₂O₅, which leads to wrong results; the \( D_p \) model use twice the coefficient of P utilization form fertilizer, once as \( C_u \) and once as \( k_p \), which also leads to wrong results.

Example of calculation:
If \( Y = 4 \) t/ha, \( C_{sp} = 18 \) kg P₂O₅/t, \( P_{ex} = Y \times C_{sp} = 4 \times 18 = 72 \text{ kg P}_2\text{O}_5/\text{ha}; G_{sa} = 3000000 \text{ kg/ha}; P = 35 \text{ ppm P}; k_P = 12 \%, \text{ CA_{pH} = 0.9, C_{Ag} = 1, then} \)

\[ P_t = 3000000 \times 35 \times 12 \times 0.9 \times 1 = 1134 \text{ kg P/ha} = 2597 \text{ kg P}_2\text{O}_5/\text{ha} \text{ (absurd)} \]
\[ D_p = (72 - 2597 \times 100) / 12 = -21363 \text{ kg P}_2\text{O}_5/\text{ha} \text{ (absurd, nonsense)} \]

With the other model [8], \( D_p = (72 - 2597) / 100 \times 12 = -21042 \text{ kg P}_2\text{O}_5/\text{ha} \text{ (absurd)}. \)

Other erroneous model [9] gives nutrient rates one hundred times smaller than normal, that means 1 kg instead of 100 kg.

The above erroneous models give absurd rates in all nutrients and all crops, not only in the exemplified P₂O₅ rate in winter wheat. Many other models published by these authors are incredible wrong.

534
CONCLUSIONS

1. Based on the TOR mathematical model, useful agrochemical tables and nomograms have been elaborated and published for the first time in order to operative settle the TOR of N, P₂O₅ and K₂O in winter wheat.

2. The agrochemical tables and nomograms can be used when operative settle of TOR are needed; the mathematical model has to be used when exact values of TOR are desired.

3. The erroneous models elaborated and published by some authors [6], [7], published also by other authors [8], [9], [10] with the same or new errors, give absurd rates in all nutrients and all crops. Such models and such books must not to be used in practice by farmers and in universities.

ACKNOWLEDGEMENTS

This paper is a tribute brought to Z. Borlan, the greatest and most original Romanian agrochemist ever, the best of the best, the mentor of the author.

REFERENCES


